Personal Test Prep

Examples of New SAT Math Questions

Number and Operations

Sequences

Geometric sequences will now appear on the test. These are sequences such as 3, 12, 48, 172,. . . Each number is 4 times the previous number. Another way of saying that is, "the sequence has a constant ratio of 4. The sequence 2, 10, 50,. . . has a constant ratio of 5. You might be asked the following:

If a sequence begins with the term 6 and has a constant ratio of *x*, the sixth term in the sequence is how many times greater than the fourth term in the sequence?

(On the test, you'll get 5 choices with a question like this, but for our purposes, we'll just solve it.)

The first term is 6, the second term is 6*x*, the third term is (6*x*)*x* or $6x^2$. Note that the third term has an exponent of 2. Likewise the fourth term has an exponent of 3 ($6x^3$). The 100th term would have an exponent of 99 ($6x^{99}$). What we need for this question is the sixth term ($6x^5$) and the fourth term ($6x^3$). To determine how many times greater the larger term is, set up a simple fraction: $\frac{6x^5}{6x^3}$. The sixes cancel out and your left with $\frac{x^5}{x^3}$. To solve, keep the base - *x* - and subtract the exponents. The solution is x^2.

Sets

A set consists of any number of elements. The union of two or more sets refers to all of their combined elements. The intersection of two or more sets refers only to the elements that they have in common. Here's a possible question:

Set A consists of all the prime numbers that are less than 15. Set B consists of all the positive multiples of 3 that are less than 15. Set C consists of all the positive factors of 30 that are less than 15. Set D consists of the intersection of Set A and Set C. What are the elements in a set that consists of the union of Set B and Set D?

(On the test, you'll get 5 choices with a question like this, but for our purposes, we’ll just solve it.)

Set A is [2, 3, 5, 7, 11]
Set B is [3, 6, 9, 12]
Set C is [1, 2, 3, 5, 6,10,]
Set D is [2,3, 5], since those are the elements that A and C have in common. The union of this set and Set B is [2,3,5,6,9,12]. That is the answer.

Algebra & Functions

Absolute Value

The absolute value of a number refers to its distance from 0 on the number line. The absolute value of 7 is 7, and the absolute value of -7 is also 7. Absolute value is always positive except that the absolute value of 0 is 0. The notation for absolute value is as follows: $|-4| = 4$. That means the absolute value of -4 is 4.
$|12 - 22| = 10$ means that the absolute value of 12 - 22 = 10.
If $|x - y| = 100$, then $x - y = 100$ or $x - y = -100$. Here’s a possible question:

If $r = -29$, and $|q + r| < 20$, what is the least possible integer value for q?

Substituting -29 for r, we get $|q - 29| < 20$. Therefore, $q - 29$ itself (NOT the absolute value of $q - 29$) is any number less than 20 AND greater than -20. For example, $q - 29$ could be -17, but it could not be - 21. In other words. . .

$$-20 < q - 29 < 20$$

Now you can simply solve for q by adding 29 to each of the three parts of the above inequality and you’ll get

$$9 < q < 49$$

Since we were asked for the least possible integer value for q, the answer is 10.

Rational Equations and Inequalities

This basically refers to solving equations that involve fractions with polynomials -- fractions with two-term expressions such as $x + 4$. For example: $\frac{x+4}{7} = \frac{x}{10}$ is a rational equation. This can be simply solved by cross-multiplying and then isolating the x. $10x + 40 = 7x$ *, which means* $3x = -40$ *so. . .* $x = \frac{-40}{3}$. This is really nothing very new for the SAT. However, be on the lookout for a solution that puts 0 into the denominator. These solutions are invalid; they must be discarded. For example.

$$\frac{x}{x-3}+\frac{1}{x-5}=\frac{-6}{x^2-8x+15}$$

To solve for x, first find the common denominator for the terms on the left so that you can add them. If you multiply these two denominators you will get a common denominator. Now we can say:

$$\frac{x}{x-3}\frac{(x-5)}{(x-5)}+\frac{1}{x-5}\frac{(x-3)}{(x-3)}=\frac{-6}{x^2-8x+15}$$

If you multiply this out, you'll get:

$$\frac{x^2-5x}{x^2-8x+15}+\frac{x-3}{x^2-8x+15}=\frac{-6}{x^2-8x+15}$$

Now that you've got the common denominator on the left (and, as it turns out, it's the SAME as the denominator on the right, but forget that for the moment), you can add the two terms on the left.

$$\frac{x^2-4x-3}{x^2-8x+15}=\frac{-6}{x^2-8x+15}$$

NOW, since the denominators on both sides of the equation are already equal, we only have to make the numerators equal. $x^2-4x-3=-6$

This should be looking like a quadratic equation to you. Since we normally solve these by getting one side to equal zero . . . $x^2-4x+3=0$

This can be factored into $(x-3)(x-1)=0$, which means that *x* = 3 or *x* = 1. HOWEVER, you must now make sure that neither of these solutions will cause 0 to be in the denominator. As it turns out, if you go back to the original equation . . .

$$\frac{x}{x-3}+\frac{1}{x-5}=\frac{-6}{x^2-8x+15}$$

. . . if *x* = 3, the first term would be zero, and that's not allowed. Therefore the only solution to this equation is the other one: *x* = 1.

Radical Equations

These are equations with square roots. Here, you must be careful not to have a negative number under the square root sign, since this would produce an undefined number. If you do end up with a solution that puts a negative number under the square root sign, you must get rid of it. For example, let's say you had to solve for *x* in this equation: $\sqrt{-x}=x+2$. Important: We can't say that there already is a problem. If *x* itself is negative, then -*x* would be positive, which is fine.

To solve, square both sides: $-x = (x+2)^2$. When multiplied out, you get: $-x = x^2 + 4x + 4$. If you add an x to both sides you'll have a quadratic equation: $x^2 + 5x + 4 = 0$. Now, factor the quadratic: $(x+4)(x+1) = 0$. This means that $x = -4$ or $x = -1$. HOWEVER, you must go back to the original equation and try these solutions out. If $x = -4$, then $\sqrt{-(-4)} = -4 + 2$, which means that $\sqrt{4} = -2$. This makes no sense; the square root of something can not be equal to -2 or negative anything. Discard $x = -4$. The only solution is $x = -1$. If we plug it in for x, we get $\sqrt{-(-1)} = -1 + 2$, which simplifies to $\sqrt{1} = 1$, which is clearly valid.

Negative and Fractional Exponents
For the first time, exponents may show up that are not positive integers. For negative integer exponents, just invert (turn upside down) the result you would have gotten if the exponent had been positive. So, 5^{-2} becomes $\frac{1}{5^2}$, which is like saying $\frac{1}{25}$. That's all there is to it. x^{-7} becomes $\frac{1}{x^7}$.

For fractional exponents, it is unlikely that you will see anything more confusing than an exponent of $\frac{1}{2}$ or $\frac{1}{3}$. An exponent of $\frac{1}{2}$ is the same as saying take the square root of something. So, $9^{\frac{1}{2}}$ is the same as $\sqrt{9}$ which is 3. For an exponent of $\frac{1}{3}$, it's the same as the cubed room of something. $27^{\frac{1}{3}} = \sqrt[3]{27}$. The answer is 3, since $3^3 = 27$.

Direct and Inverse Variation
When two variables are directly proportional, they change by the same factor. Put another way, if a and b are directly proportional then there is a constant ratio between them. In the equation $a = 5b$, the variables are directly proportional. If you doubled the value of a, you would have to double the value of b. However, in the equation $a = 5 + b$, the two variables are not directly proportional. If you doubled the value of a, you would not double the value of b.

Two variables are inversely proportional if a change in one of them causes the inverse of that change in the other. In the equation $a = \frac{3}{b}$, a and b are inversely proportional. If you doubled the value of a, you would have to halve the value of b. If you tripled the value of a, you would have to multiply the value of b by one-third.

Function, Function notation, Domain, and Range
Function: You could be given a set of paired numbers (called a relation) and asked if this relation is a function. It will be a function if each x value has only one

corresponding y value. For example, if you were given the following relation: (-3, -7), (1,1), (2,3), (3,5), you could say, yes this is a function. Each x value has only one y value. The set of all the x values is the domain and the set of all the y values is the range, so the domain is [-3, 1, 2, 3] and the range is [-7, 1, 3, 5] However, the relation (-2, 4), (0, 5), (1, 6), (-2, 7) is not a function, since -2 is paired with both 4 and 7.

You might be given a real-world situation to represent as a table or a graph of a function. For example, you could be told that a company sells greeting cards in packages of 5, 10, or 20. When the company sells a package of 5, it makes $3.00 profit; when it sells a package of 10, it makes $5.00 profit, and when it sells a package of 20, it makes $9.00 profit. The relation is (5, 3), (10, 5), (20, 9). This is a function and could be represented in a table:

Number of cards	Profit in dollars
5	3
10	5
20	9

In this chart, the domain is [5, 10, 20] and the range is [3, 5, 9].

Plotting the points (5, 3), (10, 5), and (20, 9) on the x and y coordinate system would be another way to represent the information.

<u>Function Notation</u>: $f(x) = x + 5$ is an example of function notation. It simply says that for any value x, add 5 to it. So if x is 3, $f(3) = 8$. If, on the other hand, $f(x) = 2x - 1$, then $f(1) = 2(1) - 1 = 1$, and $f(2) = 2(2) - 1 = 3$, and $f(3) = 2(3) - 1 = 5$. You can plug in any x and a different number will pop out. You could list these pairs (the x number, and the number that pops out) as a set: (1,1), (2,3), and (3,5). Furthermore, you might be asked to graph these points in a rectangular coordinate system with an x and y axis.

<u>Domain and Range</u>: You might be given something like $f(x) = \frac{x+10}{x-3}$, and then asked for the domain of this function. The domain consists of all possible x values. Since you can't have 0 in the denominator, we say that the domain of $\frac{x+10}{x-3}$ is all real numbers except 3. In general the domain would be all values for x that would not produce zero in the denominator or the square root of a negative.

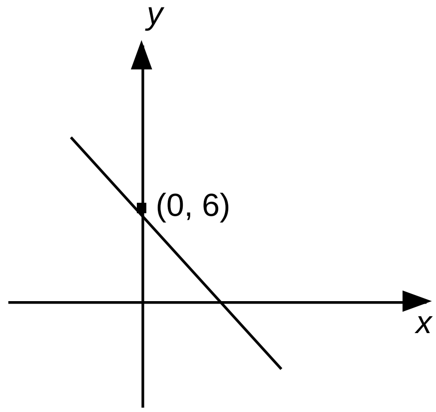
The range consists of all possible results that would "pop out" of $\frac{x+10}{x-3}$. This is a bit trickier and may not be tested. But here's how it works. If all possible results of $\frac{x+10}{x-3}$ are called y, then we can say $\frac{x+10}{x-3}=y$, and then solve for x in terms of y. If we find a value(s) for y that would result in an irrational number (zero in the denominator or the square root of a negative), the range would be all <u>other</u> values for y. So, given $\frac{x+10}{x-3}=y$, first solve for y. Cross multiply and you get $xy-3y=x+10$. Get the x terms together: $xy-x=3y+10$. Then factor the x from the two terms on the left: $x(y-1)=3y+10$. Finally, isolate the x and you get $x=\frac{3y+10}{y-1}$. Note that you can't have y = 1 because then zero would be in the denominator. Therefore, we say that the range of $\frac{x+10}{x-3}$ is all real numbers except 1.

Geometry

y-intercept

This will not be new for some students. The standard equation for a line is $y=mx+b$. What that means is that everything that can be said about a line can be said using that equation. The letter m refers to the slope of the line; the letter b is the y-intercept, which simply refers to the point on the y axis where the line crosses it. The values for m and b never change for any line. The variables x and y refer to the coordinates for any point on that particular line. So you might see a line described with the equation $y=3x+2$. In this case, the slope is 3 and the y-intercept is 2. That means that the line crosses the y axis at the point (0,2). If you were told that the point (-1, y) was on this line, you could find the y value for this point by using the equation. Since the x value is -1, you would plug that into the equation and solve for y. Thus, we have $y=3(-1)+2$, which means that $y=-1$. The point $(-1,-1)$ must lie on this line.

In the diagram below, just about all we know about the line that's drawn is that it has a y-intercept of 6. (We can, however, also say that it has a negative slope since it travels down to the right. If it went up as it traveled from left to right, we would say that it has a positive slope. If a line is parallel to the x axis, it has a slope of 0. If it is parallel to the y axis, the slope is not defined.) So, knowing that the y-intercept is 6, you could come up with the equation: $y=mx+6$. Now, if you're told that the point (4,1) is on the line, you could say that $1=4m+6$. You've now got an equation that allows you to solve for m, the line's slope. Isolate m and you'll get $m=-\frac{5}{4}$.



Geometric Notation

You may be familiar with the following symbols, but they're new to the SAT.

$\overleftrightarrow{AB}$ refers to a straight line passing through points *A* and *B*.

$\overrightarrow{AB}$ refers to a ray, starting at *A* and passing through *B*.

$\overline{AB}$ refers to a line segment with end points at *A* and *B*. AB is the distance of this segment.

$ABC \cong CDE$ means that two shapes (in this case, triangles) are **congruent**.

Solving With Trigonometry

The new SAT will include problems that can be solved using basic trigonometry, but this has always been the case. No problem should **have** to be solved with trigonometry. All students should know the ratios of the sides of 30-60-90 and 45-45-90 triangles. They are, respectively, $1:\sqrt{3}:2$ and $1:1:\sqrt{2}$. Therefore, if you know that the hypotenuse of a 30-60-90 triangle is 10, you would know, without any real calculations, that the short leg (across from the 30-degree angle) is 5, and the longer leg (across from the 60-degree angle) is $5\sqrt{3}$. With a 45-45-90 triangle -- also known as an isosceles right triangle -- if the legs each measure 3, then the hypotenuse is $3\sqrt{2}$.

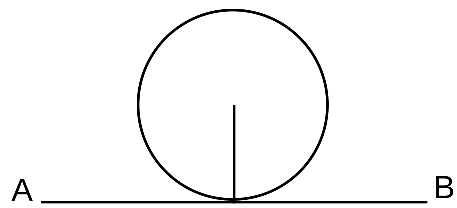
The Midpoint and Distance Formulas

To find the **midpoint** between points *A* and *B*, take the average of the *x* coordinates and the average of the *y* coordinates. For example, if point *A* has coordinates of (3, -7) and point *B* has coordinates of (1, -1), the coordinates of the midpoint of $\overline{AB}$ are $\frac{3+1}{2}$ and $\frac{-7+(-1)}{2}$, which simplifies to (2,-4).

To find the **distance** between points A and B using their coordinates, you can apply the following formula: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Using the coordinates in the previous example, we get $\sqrt{(3-1)^2 + (-7-(-1))^2}$, which simplifies to $\sqrt{4+36}$, which further simplifies to $\sqrt{40}$. This can be rewritten as $2\sqrt{10}$.

Properties of Tangent Lines

The new SAT may test the following: A line tangent to a circle is perpendicular to a radius drawn to the point of tangency. Therefore, in the circle below, we know that the radius shown forms a right angle with $\overline{AB}$.

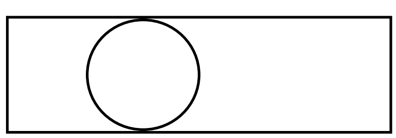


Transformations

You might now be tested on the effect that a transformation will have on a graph of a function. For example, given the graph of the function $f(x)$, you would need to know that the graph of $f(x)-4$ would have the same shape but is moved "down" 4 units. The graph of $f(x)+4$ would be the same shape moved "up" 4 units. The graph of $f(x-4)$ -- note the parentheses -- would look the same as $f(x)$ but shifted 4 units to the right. Naturally, $f(x+4)$ is shifted 4 units to the left. The graph of $-f(x)$ has the same shape of $f(x)$, but turned upside down. It is reflected about the x axis. The graph of $f(-x)$ is the mirror image of $f(x)$. It is reflected about the y axis.

Geometric Probability

This is not an entirely new concept. The basic probability formula still applies: Probability = The number of desired outcomes/Total number of outcomes. For example, the following diagram shows a circle inscribed in a 3 foot x 10 foot rectangle, drawn to scale. If a penny was thrown onto this shape, without being aimed, what is the probability that the penny would land on the circle?



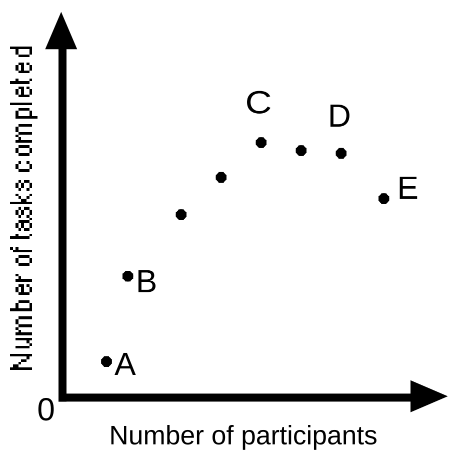
First, we can determine that the rectangle has an area of 30 square feet (3x10). Since the circle is inscribed in the rectangle, it has a diameter of 3 and a radius of 1.5 or $\frac{3}{2}$. Therefore, the area of the circle is $\frac{9\pi}{4}$ square feet (πr^2). The odds (probability) of the penny landing on the circle is $\frac{\frac{9\pi}{4}}{30}$, which simplifies to $\frac{9\pi}{4} \times \frac{1}{30}$, which simplifies further to $\frac{3\pi}{40}$. To convert this to a percentage, multiply by 100 and then stick a % sign on it. So, $\frac{3\pi}{40} \times 100 = \frac{15\pi}{2}\%$. There is a $\frac{15\pi}{2}\%$ chance of landing the penny on the circle.

Data Analysis

Graphs and charts

Analyzing graphs and charts has always been a part of the SAT, but some related terms and concepts might show up for the first time. Being able to read a scatterplot is an important skill to have as illustrated below.

In an experiment on worker productivity, project teams of different sizes were given the same tasks to complete. The number of tasks completed was then plotted against the number of participants in the group. In the sketch below, which of the labeled points represents the group that completed the most tasks per number of participants?



The correct answer is point B. This is the point that has the largest task/participant ratio. You can think of it this way: If you drew a line from 0 to point B, the slope of this line would be greater than the slope of any line drawn from 0 to a different point.

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